Optimizing Data Structures in High-Level Programs:

New Directions for Extensible Compilers based on Staging

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How should we build compilers?
Principles of Compiler Design

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Programs and Languages

Productivity: Generalization, Abstraction

Hardware

Performance: Specialization, Concretization
abstract class Vector[T:Numeric] {
    val data: Array[T]
    def +(that: Vector[T]) =
        Vector.fromArray(data.zipWith(that.data)(_ + _))
}

object Vector {
    def fromArray[T:Numeric](a: Array[T]) =
        new Vector { val data = a }
    def zeros[T:Numeric](n: Int) =
        Vector.fromArray(Array.fill(n)(i => zero[T]))
}

abstract class Matrix[T:Numeric] { ... }

case class Complex(re: Double, im: Double) {
    def +(that: Complex) = Complex(re + that.re, im + that.im)
    def *(that: Complex) = ...
}

implicit object ComplexIsNumeric extends Numeric[Complex] { ... }
User Program

def diag(k: Int, n: Int) =
    k * Matrix.identity(n)

val m1 = (v1+v2).trans * (v1+v2)
val m2 = diag(2, m1.numRows)

if (scale) println(m1*m2)
else println(m1)

Elegant and high level, but is it fast?
The compiler / VM will figure out how to run it fast

(wishful thinking)
No it doesn’t:
10 to 100x slower than optimized code with arrays and loops!

(hard reality)
Many productivity features don’t perform well
• Problem 1: abstraction penalty

• Problem 2: compiler lacks semantic knowledge
abstract class Vector[T:Numeric] {
    val data: Array[T]
    def +(that: Vector[T]) =
        Vector.fromArray(data.zipWith(that.data)(_ + _))
}

object Vector {
    def fromArray[T:Numeric](a: Array[T]) =
        new Vector { val data = a }
    def zeros[T:Numeric](n: Int) =
        Vector.fromArray(Array.fill(n)(i => zero[T]))
}

abstract class Matrix[T:Numeric] { ... }

case class Complex(re: Double, im: Double) {
    def +(that: Complex) = Complex(re + that.re, im + that.im)
    def *(that: Complex) = ...
}

implicit object ComplexIsNumeric extends Numeric[Complex] { ... }
Idea: Let’s use Macros or Staging!

compose program fragments
programmatically remove abstraction
Lightweight Modular Staging (LMS)

• Use a type constructor Rep[T] to delay evaluation of expressions to the next (generated) stage
• Lift operations from type T to type Rep[T], generating code to apply the operation later
• Expressions of type T are evaluated immediately and become constants in generated code
• Maintain evaluation order within a stage (unlike syntactic quasi-quotation)

Example: Vectors

abstract class Vector[T:Numeric] {
  val data: Array[T]
  def +(that: Vector[T]) =
    Vector.fromArray(data.zipWith(that.data)(_ + _))
}

object Vector {
  def fromArray[T:Numeric](a: Array[T]) =
    new Vector { val data = a }

  def zeros[T:Numeric](n: Int) =
    Vector.fromArray(Array.fill(n)(i => zero[T]))
}
abstract class Vector[T:Numeric] {
  val data: Rep[Array[T]]
  def +(that: Vector[T]) =
    Vector.fromArray(data.zipWith(that.data)(_ + _))
}

object Vector {
  def fromArray[T:Numeric](a: Rep[Array[T]]) =
    new Vector { val data = a }

  def zeros[T:Numeric](n: Rep[Int]) =
    Vector.fromArray(Array.fill(n)(i => zero[T]))
}
Example: Array

```scala
implicit class ArrayOps[T](a: Rep[Array[T]]) {

  def zipWith[U,V](b: Rep[Array[U]])(f: (Rep[T],Rep[U]) => Rep[V]) = 
  Array.fill(min(a.length,b.length))(i => f(a(i), b(i))
}

object Array {
  def fill[T](n: Size)(f: Rep[Int] => Rep[T]) = {
    val r = NewArray[T](n)
    var i: Rep[Int] = 0     // staged variable
    while (i < n) {         // staged loop
      r(i) = f(i)
      i += 1
    }
    r
  }
}
```
Example: Matrix

```kotlin
val m = Matrix.rand(500, 100)
val n = Matrix.rand(100, 500)
m * n
```
Victory?
• Problem 1: abstraction penalty
  – Staging

• Problem 2: compiler lacks semantic knowledge
def diag(k: Int, n: Int) = k * Matrix.identity(n)

val m1 = (v1+v2).trans * (v1+v2)
val m2 = diag(2, m1.numRows)

if (scale) println(m1*m2) // m1*(k*id) = k*m1*id = k*m1
else println(m1) // no need to compute m2
Limitations of Staging / Macros

• Want to treat matrices as symbolic entities with algebraic laws
• \texttt{m*ident} expanded into arrays / loops before reaching the compiler
  – Too late to perform symbolic simplification!
Extend compiler with high-level semantic knowledge
Extensible Compilers

• Vector/Matrix operations as IR nodes
• Optimization pass to simplify \( m \times \text{ident} \Rightarrow m \)
• Another pass to expand operations into loops
• Usual limitations:
  – heavyweight
  – IR-to-IR transformers much lower level, harder to express than with macros / staging
  – Phase ordering problems between new and existing optimizations
• Problem 1: abstraction penalty
  – Staging

• Problem 2: compiler lacks semantic knowledge
  – Extensible compilers

• Neither solution alone is sufficient!
Use staging in intermediate languages!
Stage away abstractions *after* applying symbolic rewrites, CSE, etc!
A staged interpreter is a program transformer

Instead of Tree => Tree:
Tree => staged code that computes a Tree
Not all Transformations are Alike

• **Lowerings**
  – e.g., vector/matrix ops \rightarrow loops over arrays
  – Have a natural ordering
  – Can be profitably arranged in separate passes
  – Easy to solve with staged interpreters

• **Optimizations**
  – No clear ordering, prone to phase ordering problems
  – Must be combined for maximum effectiveness (optimistic assumptions)
  – Should be applied exhaustively before lowering takes place

• **Should optimize, lower, optimize, lower, ...**
  until lowest-level representation is reached
How to combine optimizations?

Rewriting using smart constructors for IR nodes:
The only problem is loops
Speculative Rewriting

- Apply all possible transformations optimistically
  - Ignore loop-carried dependencies, etc.
- If an assumption is violated, throw away transformed result and start again
- Repeat until fixed point is reached

```javascript
var x = 7
var c = 0

while (c < 10) {
  if (x < 10) print("!")
  else x = c
  print(x)
  print(c)
  c += 1
}
```

See: Lerner, Grove, Chambers (POPL’2002); Supercompilation: Turchin, Klimov, ....
Example: Matrix

trait MatrixExp extends BaseExp {
  trait Matrix[T]

  case class MatrixTimes[T:Numeric](a: Rep[Matrix[T]], b: Rep[Matrix[T]])
    extends Def[Matrix[T]]
    extends Def[Matrix[T]]

  def infix_*[T:Numeric](a: Rep[Matrix[T]], b: Rep[Matrix[T]]) =
    reflect(MatrixTimes(a, b))
    reflect(MatrixPlus(a, b))
}

trait MatrixExpOpt extends MatrixExp {
    (a, b) match {
      case (Def(MatrixTimes(a1, b)), Def(MatrixTimes(a2, c))) if a1 == a2 =>
        a1 * (b + c) // A*B+A*C => A*(B+C)
      case _ => super.infix_+(a, b)
    }
}
Example: Matrix

trait MatrixExpLower extends MatrixExp {

  def matrixTimesImpl[T](a: Rep[Matrix[T]], b: Rep[Matrix[T]]) = {
    val res = MatrixNew(a.rows, b.cols)
    for (i <- 0 until a.rows) {
      for (j <- 0 until b.cols) {
        for (k <- 0 until a.rows) {
          res(i, j) += a(i, k) * b(k, j)
        }
      }
      res
    }

  override def onCreate[T](sym: Rep[T], rhs: Def[T]) = rhs match {
    case MatrixTimes(a,b) => atPhase(lowering) { matrixTimesImpl(a,b) }
    case _ => super.onCreate(sym,rhs)
  }
}
What we have achieved:

• CSE, DCE on matrix operations done by LMS-Core compiler
• Added custom rewrite: $A*B+A*C \Rightarrow A*(B+C)$
  – Rewrites compose!
• Added custom lowering: MatrixTimes => loops
  – Implemented as a staged method
• Uniform low-level loop abstraction
  – fusion and data parallelism
def square(x: Rep[Double]) = x*x

def mean(xs: Rep[Vector[Double]]) =
    xs.sum / xs.length

def variance(xs: Rep[Vector[Double]]) =
    xs.map(square) / xs.length - square(mean(xs))

val v1 = Vector.fill(n) { i => 1 }
val v2 = Vector.fill(n) { i => 2*i }
val v3 = Vector.fill(n) { i => v1(i) + v2(i) }

val m = mean(array3)
val v = variance(array3)

println(m)
println(v)

// begin reduce x47,x51,x11
var x47 = 0
var x51 = 0
var x11 = 0
while (x11 < x0) {
    val x44 = 2.0*x11
    val x45 = 1.0+x44
    val x50 = x45*x45
    x47 += x45
    x51 += x50
    x11 += 1
}

// end reduce
val x48 = x47/x0
val x49 = println(x48)
val x52 = x51/x0
val x53 = x48*x48
val x54 = x52-x53
val x55 = println(x54)

3+1+(1+1) = 6 traversals, 4 arrays
1 traversal, 0 arrays
Evaluation
def preferences(ratings: Rep[Matrix[Int]], sims: Rep[Matrix[Double]]) = {
    sims.mapRowsToVector { testProfile =>
        val num = sum(\(0\), ratings.numRows) {
            i => testProfile(ratings(i,1)) * ratings(i,2)
        }
        val den = sum(\(0\), ratings.numRows) {
            i => abs(testProfile(ratings(i,1)))
        }
        num / (den + 1)
    }
}
Regular Expressions

```scala
def convertNFAtoDFA(flag: Boolean, state: NIO): DIO = {
  val cstate = canonicalize(state)
  dfa_trans(flag) { c: Rep[Char] => exploreNFA(cstate, c) {
    convertNFAtoDFA
  }
}
}
```

**Figure 13.** Regexp Benchmark. The first graph shows the relative execution time of matching a respectively 10, 100, $10^7$ long input string of the form A+B on the regular expression .AAB. The second graph summarizes the relative performance over many different inputs and regular expressions.
Collections and Queries

// lineItems: Array[LineItem]
val q = lineItems filter (_._l_shipdate <= Date('19981201')).groupBy (_._l_linenestatus) map {
  case (key,g) => new Record {
    val lineStatus = g.key
    val sumQty = g.map(_._l_quantity).sum
    val sumDiscountedPrice = g.map(r => r._l_extendedprice*(1.0-r._l_discount)).sum
    val avgPrice = g.map(_._l_extendedprice).sum / g.size
    val countOrder = g.size
  }
} sortBy(_._lineStatus)
String Templating

```scala
def link(uri: Rep[String], name: Rep[String]) =
  List("<a href='", uri, "'>", name, "</a")

def renderItem(i: Rep[Item]) = List("<li><ul>") ++
  i.subitems.flatMap(link(i.name, i.link)) ++ List("</ul></li>")

def usersTable(items: Rep[List[Item]]) = List("<ul>") ++
  items.flatMap(renderItem) ++ List("</ul>")
```

---

**Normalized Exec Time**

<table>
<thead>
<tr>
<th>HTML Template</th>
<th>Depth 3</th>
<th>Depth 4</th>
<th>Depth 5</th>
<th>Depth 6</th>
<th>imdb.com</th>
<th>cnn.com</th>
<th>dell.com</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plain Scala</td>
<td>1.2</td>
<td>1.2</td>
<td>1.1</td>
<td>1.1</td>
<td>1.4</td>
<td>1.4</td>
<td>1.4</td>
</tr>
<tr>
<td>Staged</td>
<td>5.7</td>
<td>7.2</td>
<td>8.1</td>
<td>10.4</td>
<td>8.7</td>
<td>6.2</td>
<td>7.1</td>
</tr>
<tr>
<td>Staged + Fusion</td>
<td>0.2</td>
<td>0.4</td>
<td>0.6</td>
<td>0.8</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Evaluation Summary

Order of magnitude speedups on a variety of high-level programs by:

• fusing collection operations
• changing data layout
• applying (generic and specific) optimizations on high-level objects
Key Take-Aways:

1. Compilers need to make sense of high-level, domain-specific abstractions.

2. Many different techniques (staging, extensibility, speculative rewriting, fusion): We really need to combine all of them to achieve good results!

scala-lms.github.com
Backup Slides
Key Take-Aways:

• Optimizations should be combined
  – Avoid pessimistic assumptions
  – Avoid phase ordering problems
  – Speculative rewriting: generic solution for forward DF

• Lowering transforms should be separate passes
  – Apply high-level optimizations exhaustively before switching representations (e.g. Matrix/Vector to arrays and loops)

• Staged IR interpreters as IR to IR transformers:
  – Programmatically remove abstraction overhead at all intermediate stages
  – Simplify implementation
Expression Templates

- Purely frontend approach
- Not integrated with DCE, CSE
- Optimization horizon restricted to extent of compound expression
Rewriting Frameworks

- Graphs vs trees
- Dependency information
- “model transformation all the way down” similar to our approach to lowering
- But we want to combine optimizations, not layer them
- Interpretation is simpler than transformation!
Fusion

• Horizontal and vertical
• Includes flatMap and groupBy
• Not restricted to scope of single expression; only one resulting loop here:

```scala
def calcSum() = array.sum
def calcCount() = array.filter(_ > 0).count
println("sum: " + calcSum())
println("avg: " + (calcSum() / calcCount()))
```
Lisp/Scheme

• Also pervasive use of macros in compilation

• Which implementation:
  – Supports an open set of algebraic rewrites for vector/matrix operations without phase ordering problems?
  – Reuses generic CSE, DCE etc on vectors and matrices?
  – Can apply AOS to SOA transforms?
Partial Evaluation of Interpreters

• Earlier work on program transformation by partial evaluation
• Different techniques
• Arbitrary compiler optimisations, not just constant folding
• Arbitrary computation at staging/specialization time to remove abstraction overhead
• Strong guarantees about shape of residual code (Rep[T] vs T types)
EOF